**TASK 8: HOW MUCH MUST I KNOW?**

**In-class Investigation**

**Unit 3**

**Topic 3.3: Discrete random variables**

**Course-related information**

The concepts and skills included in this investigation relate to the following dot points within the WA Mathematics Methods syllabus.

3.3.13 examine the concept of Bernoulli trials and the concept of a binomial random variable as the number of ‘successes’ in independent Bernoulli trials, with the same probability of success in each trial

3.3.14 identify contexts suitable for modelling by binomial random variables

3.3.15 determine and use the probabilities associated with the binomial distribution with parameters and ; note the mean and variance of a binomial distribution

3.3.16 use binomial distributions and associated probabilities to solve practical problems

The ability to choose and use appropriate technology to enhance and extend concept development is also incorporated within some of the items.

**Background information**

Students should be able to use their calculators to evaluate binomial discrete and cumulative probabilities.

**Task conditions**

The task should be done under test conditions in class.

**How much must I know?**

**In-class investigation Total: 27 marks**

**NB Much of the work required in this investigation can be done on a graphical/CAS calculator. You must be sure to include enough working so that your method as well as your answer can be checked.**

Penny and Bob know that Test 1, the first assessment test for the term in Chemistry, is scheduled for next week. Penny has studied consistently but only knows about 60% of the topic very well, so she expects to obtain 60% of the answers correct in the test. However, she is aiming to obtain a minimum of 80% of the answers correct in order to gain an A grade. Bob has done some study and he thinks that he knows about 40% of the required material.

The students have been told that Test 1 will be a multiple choice test consisting of 10 questions, each with a choice of 5 answers, so the probability of getting an answer correct by guessing is 0.2.

Assume that if 60% of the content being tested is known, then 60% of the questions will be correct and guessing is not required.

**Question 1** **(9 marks)**

If Penny and Bob sat Test 1 today and guessed the answers they did not know,

(a) what is the expected number of correct answers that would be obtained by

(i) Penny? (3)

(ii) Bob? (3)

(b) what is the probability that Penny would score 80%? (1)

(c) what is the probability that Bob passed the test (ie. he obtained a minimum

of 50%)? (2)

In the week prior to Test 1, Penny studied hard and was certain she knew 80% of the topic well. Bob did sufficient study to feel confident that he knew well 50% of the topic.

**Question 2** **(9 marks)**

Test 1 consisted of 10 multiple choice questions, each with a choice of 5 answers.

Penny and Bob sat Test 1 and guessed the answers they did not know.

(a) What is the probability that Penny obtained

(i) 90%? (1)

(ii) 100% (1)

(b) What is the probability that Bob

(i) passed (obtained a minimum of 50%)? (1)

(ii) achieved a higher score than Penny if she obtained 80%? (3)

(c) What is the probability that Penny and Bob both scored 90%? (3)

A few weeks later, Bob and Penny sat for Test 2, the next assessment test in Chemistry.

Test 2 was also a multiple choice test but consisted of 20 questions, with a choice of 4 answers per question.

Question **3** **(9 marks)**

Investigate the differences in the probabilities of achieving

a pass (50% or higher), an A grade on the test (80% or higher)

or a perfect score (100%) in Test 1 and Test 2 for Penny and Bob.

Test 1 had 10 questions with a choice of 5 answers per question.

Test 2 had 20 questions with a choice of 4 answers per question

Assume that Penny knew 80% of the work thoroughly before each test and that

Bob knew 50% of each topic well.

Comment on the differences identified and give possible reasons for these differences.

End of questions

**How much must I know?**

**In-class investigation**

**Solutions and marking key**

**Question 1(a)**

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| Solution | |
| *E(X) = np*  (i) Penny knew 60% so she only had to guess 40% of the questions  *n* = 4, *p* = 0.2  Penny : *np =* 4 × 0.2 = 0.8  ∴ Expected number of correct answers for Penny = 6 + 0.8 ≈ 7  (ii) Bob knew 40% so he only had to guess 60% of the questions  *n* = 6, *p* = 0.2  Bob : *np =* 6 × 0.2 = 1.2  ∴ Expected number of correct answers for Bob = 4 + 1.2 ≈ 5 | |
| Mathematical behaviours | Marks |
| (i)   * determines the correct value for *n* for Penny * determines the correct *np* value for Penny * calculates correctly the expected number of questions for Penny   (ii)   * determines the correct value of *n* for Bob * determines the correct *np* value for Bob * calculates correctly the expected number of questions for Bob | 1  1  1  1  1  1 |

**Question 1(b)**

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| Solution | |
| Penny knew 60% so she only had to guess 40% of the questions  *n* = 4, *p* = 0.2  P(80%) = ?  P(x = 2) =  P(80%) = 0.1536 | |
| Mathematical behaviours | Marks |
| * calculates P(x = 2) correctly | 1 |

**Question 1(c)**

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| Solution | |
| Bob knew 40% so he only had to guess 60% of the questions  *n* = 6, *p* = 0.2  P(a minimum of 50%) = P(x = 1,2,3,4,5,6) = P(x >1)  = 1 – P(x = 0)  = 1 – 0.2621  = 0.7379 | |
| Mathematical behaviours | Marks |
| * identifies the possible scores * calculates P(x > 1) correctly | 1  1 |

**Question 2(a)**

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| Solution | |
| Penny knew 80% so she only had to guess 20% of the questions  *n* = 2, *p* = 0.2  (i) P(x = 1) =  P(90%) = 0.32  (ii) P(x = 2) =  P(100%) = 0.04 | |
| Mathematical behaviours | Marks |
| * (i) calculates P(x = 1) correctly * (ii) calculates P(x = 2) correctly | 1  1 |

**Question 2(b)**

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| Solution |
| Bob knew 50% so he only had to guess 50% of the questions  *n* = 5, *p* = 0.2  (i) P(pass) = ?  P(x > 0) = 1  P(pass) = 1  (ii) P(Higher than Penny’s score of 80%) = P(90% or100%)  P(a minimum of 90%) = P(x = 4, 5) = P(x = 4) + P(x = 5)  = 0.0064 + 0.00032  = 0.0067  OR  P(a minimum of 90%) = P(x = 4, 5) = P(x > 4)  = 1- P(x< 3)  = 0.0067 |

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| Mathematical behaviours | Marks |
| * (i) recognises probability is 1 or calculates P(x > 0) * (ii) identifies possible scores * calculates P(4) or P(5) or P(x< 3) correctly * calculates P(x = 4, 5) correctly | 1  1  1  1 |

**Question 2(c)**

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| Solution | |
| Penny: P(90%) = 0.32 ( calculated already in question 2(a)(i))  Bob:  *n* = 5, *p* = 0.2  P(90%) = P(x = 4)=  Need not show method as may have calculated P(90%) in question 2(b)(ii)  Because Penny’s score is independent of Bob’s score,  P(both Penny and Bob score 90%) = 0.32 × 0.0064 = 0.002048 | |
| Mathematical behaviours | Marks |
| * determines P(score = 90%) correctly for Bob * identifies independence of the test scores * calculates correct probability for P(Penny and Bob) | 1  1  1 |

**Question 3**

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| Solution |
| **Test 1** 10 questions, 5 answers  Penny knew 80% so she only had to guess 20% of the questions  *n* = 2, *p* = 0.2  Bob knew 50% so he only had to guess 50% of the questions  *n* = 5, *p* = 0.2  **Test 2** 20 questions, 4 answers  Penny knew 80% so only had to guess 20% of the questions  *n* = 4, *p* = 0.25  Bob knew 50% so only had to guess 50% of the questions  *n* = 10, *p* = 0.25  **Penny:**  **Test 1**  **Test 2**  P(Score > 50%) = 1  P(Score > 80%) = 1  P(Score = 100%)  Use n = 4, p = 0.25  P(x = 4) =  P(Score = 100%) = 0.004  P(Score > 50%) = 1  P(Score > 80%) = 1  P(Score = 100%)  Use n = 2, p = 0.2  P(x = 2) =  P(Score = 100%) = 0.04  **Bob:**  **Test 2**  P(Score > 50%) = 1  P(Score > 80%)  Use n = 10, p = 0.25  P(x > 6) =  P(Score = 100%) =  **Test 1**  P(Score > 50%) = 1  P(Score > 80%)  Use n = 5, p = 0.2  P(x > 3) =  P(Score = 100%)  =  = 0.00032 |

**Question 3(cont’d)**

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| Solution | |
| The probability of guessing a correct answer for Test 2 is 0.25 which > 0.2, the  probability of guessing a correct answer in Test 1.  So if both tests had had the same number of questions, the chance of achieving a higher score by guessing would be greater with Test 2.  The chance of Penny achieving 100% and P(score > 80%) for Bob is greater with Test 1. Test 2 has 20 questions and Test 1 has 10.  So it would seem that the probability of achieving a particular score by guessing is influenced by the length of the test. | |
| Mathematical behaviours | Marks |
| * calculates correctly P(score =100%) Test 2 for Penny * calculates correctly P(score > 80%) Test 1 for Bob * identifies *n*=10, *p*=0.25 for Test 2 for Bob * calculates correctly P(score > 80%) Test 2 for Bob * calculates correctly P(score = 100%) Test 1 for Bob * calculates correctly P(score = 100%) Test 2 for Bob * comments on the probabilities of obtaining a correct answer in each test * comments on the number of questions in each test * compares probabilities of achieving scores obtained by Penny and by Bob and makes sensible conclusions | 1  1  1  1  1  1  1  1  1 |